Tutorial 3 for MATH 2020A (2024 Fall)

1. Let $R \subset \mathbb{R}^2$ be the region cut by the curve $r = 2(2 - \sin(2\theta))^{\frac{1}{2}}$ (in polar coordinate) in the first quadrant. Find the area of R.

> **Solution:** $2\pi - 2$

2. Let P_0 be a point inside a disk of radius a and let h denote the distance from P_0 to the center of the disk. Let d denote the distance from an arbitrary point P to P_0. Use only the quantity a and h to represent:

   (a) **The average value of d over the disk** when P_0 is the center of the disk.

   (b) **The average value of d^2 over the disk** when P_0 is an arbitrary point inside the disk.

> **Solution:** (a)$\frac{2}{3}a$; (b)$\frac{a^2}{2} + h^2$

3. Let $\Omega \subset \mathbb{R}^3$ be the noncircular right cylinder whose base $R \subset \mathbb{R}^2$ lies in the $xy - plane$ inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$ (both in polar coordinate), and whose top lies in the plane $z = x$. Sketch R and find the volume of Ω.

> **Solution:** $\frac{5}{8}\pi + \frac{4}{3}$

4. Consider the function $f(x, y) = \frac{1}{1-x^2-y^2}$ and regions $R_1 = \{(x, y) : x^2 + y^2 \leq \frac{3}{4}\}$, $R_2 = \{(x, y) : x^2 + y^2 \leq 1\}$.

   (a) Evaluate the double integral $\iint_{R_1} f(x, y)\, \mathrm{d}A$.

   (b) Does the double integral $\iint_{R_2} f(x, y)\, \mathrm{d}A$ exist? Why?

> **Solution:** (a)$2\pi \ln 2$; (b)Does not exist, prove by showing the Riemann sum cannot converge.